PREPARING TO TEACH GRADE 12 FINANCIAL MATHEMATICS: A CASE STUDY OF STUDENT TEACHERS' LEARNING EXPERIENCES

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This paper focuses on identifying the errors that second year student teachers, majoring in mathematics, make and on understanding the misconceptions that they have with regard to grade 12 financial mathematics. Data from seventy students' responses to grade 12 financial mathematics examination tasks were collected and analysed, after which twelve of these students were interviewed about the mathematical reasoning that they used. Our findings indicate that students' errors and misconceptions on this topic are conceptual, procedural and linguistic. We argue that it is critical for students to be familiar with the geometric sequence - the map of financial mathematics. Also, we argue that financial mathematics terms need to be thoroughly taught because many student errors on this topic also stem from misunderstandings of terminology.

INTRODUCTION

This article is about mathematics major student teachers' errors and misconceptions in the area of financial mathematics. Using Davis' (1984) frames as knowledge representation structures, and Tall and Vinner's (1981) concept definition and concept image notions, grade 12 financial mathematics tasks where given to second year mathematics major students. This was done to determine the errors that students still held on this topic even though they were now at university. Such as study is seen by the authors of this paper, who are the lecturers of these students as a methodology of informing good preparation for teaching our student teachers.

Financial mathematics is not only an interesting mathematics topic in its own right, but it is an aspect of mathematics that has applications in daily life, for everyone. Yet many people do not understand even the basics of financial mathematics. Mathematics major student teachers who do not understand this topic often fail one of their maths courses. In this article we argue that exploring student teachers' errors and misconceptions in financial mathematics can help researchers and teacher educators to identify key epistemological factors which could help stakeholders to better handle the topic.

Exploring student's epistemological difficulties through error analysis and diagnosis is an essential component of quality teaching (Makonye & Luneta, 2013; Nesher, 1987). It is important because it informs researchers, teachers and students themselves about the difficulties on specific mathematical objects that learners encounter.

Errors provide educators insight in to learners' current knowledge that can be used as a resource for teaching (Gallagher, 2004; Borasi, 1994). Errors can reveal to teachers exactly what learners think about certain mathematical work. Shulman (1986), argued that the teacher's knowledge of the likely errors and misconceptions that learners are prone to, as individuals or as groups, when they encounter particular mathematical concepts constitute Pedagogical Content Knowledge (PCK). Shulman concluded that educators would not be effective teachers if they were not knowledgeable of the errors and misconceptions learners usually held on particular concepts. A teacher who is aware of the likely misconceptions learners might have on a class of mathematics concepts is empowered. He/she is likely to be on the lookout for them. Also such a teacher can hypothesise situations that induce learners to elicit those errors; what Smith et al. (1993) referred to as 'confronting students with their errors'. Once such learners voice their misconceptions in the open, educators can devise pedagogic approaches to help learners see that their point of view is in fact not good enough. For these reasons, it is important for mathematics teacher educators and the student teachers with whom they work to determine the errors and misconceptions that cause difficulties for these students.

Over the last three decades, many papers have been written on the misconceptions that school children harbour when learning arithmetic, algebra, geometry, statistics and probability to name a few. These papers report that the errors exhibited, whether shared or idiosyncratic follow carefully reasoned patterns and are quite predictable, provided one have understood them. However there have been very few papers written to diagnose the errors and misconceptions that the student teachers and teachers themselves have in the mathematics content that they teach children. Such teacher errors and misconceptions are quite problematic because the teachers inadvertently pass on wrong ideas to the learners, which they then take up as knowledge. One important area in which teachers' exhibit errors and misconceptions is financial mathematics.

AIM

The research aims to explore the errors that mathematics major students have on grade 12 financial mathematics. It also aims to explore what frames of reasoning induce them to have those errors.

This paper focuses on identifying the errors that second year mathematics major students have on grade 12 financial mathematics. It aims investigate the reasons why students hold on to these errors on grade 12 financial mathematics.

SIGNIFICANCE OF THE RESEARCH

Since the onset of democracy, the average performance of South African students on periodic international comparative mathematics tests have been consistently under expectation (see Howie, 2003; Reddy, Winnaar, Visser, Arends, Mthethwa, Juan, Rogers, Feza-Piyose & Prinsloo, 2013).). One would suspect that such attainment on the part of learners might not be wholly their fault. It might be that some of the teachers who teach these students are not necessarily more-knowledgeable others in Vygotskian terms. Indeed mathematics teachers who mark matriculation examination scripts were recently themselves requested to write their learners' grade 12 mathematics examinations. The results were not surprising in that about 60% of the teachers could not pass the examinations written by their own learners. At our own school of education fourth year students about to graduate also dismally failed in a similar examination. This shows that the mathematics content knowledge for our mathematics teachers cannot be taken for granted. It is imperative that there be research which explores the content knowledge of mathematics student teachers. This is important in that prospective teachers' knowledge deficit can be identified, understood so that these can be rectified before the students graduate. Since student teachers bring to higher education the mathematical errors they hold from high school, a research which brings to light such errors in important one as it helps to ensure that teachers who graduate do not have embarrassing deficits in the content knowledge of the mathematics they will teach.

THEORETICAL FRAMEWORK

The research is studied through constructivism and particularly through Davis' (1984) work on frames that learners construct and use to learn and do mathematics. Constructivists argue that learners are not explicitly taught the errors and misconceptions they have (see for instance Confrey & & Kazak, 2006; Davis, 1984; Smith et al., 1993). Student errors and misconceptions are their current knowledge which they use to connect to new knowledge. If new knowledge is connected to current knowledge that is incorrect unfortunately another error will occur. Also even if current knowledge is correct, errors can result of the current correct knowledge is being connected to new knowledge. The new knowledge might not be portable with current knowledge and the learner is mistaken that it is. Definitely new knowledge cannot sit in a vacuum, it holds on to something however tenuous a learner previously learnt.

Some education theories such as behaviourism (Skinner, Pavlov, & Thorndike, as cited in Todes, 2002; McLeod, 2007) viewed errors as undesirable in the learning process. Their stance was that errors must be punished and weeded out so that they become extinct. On the other end of the spectrum, constructivists view errors as most useful resources in teaching and learning.

They see errors as learners' attempt to construct meaning in a learning context. To constructivists, once a learner shows an error, the teacher must be strategic about it. He/she must refrain from immediately supplying a correct answer as it would be missing an opportunity. Rather, the teacher helps the learner to reconsider his answer by requesting the learner to compare his answer with that of one or two other colleagues. That way, that learner encounters peer induced cognitive conflict that could help her to realize her error.

So learners build misconceptions by themselves as they strive to interpret new experiences and give meaning to them. According to Davis (1984), student's errors are not random; on the contrary, they turn out to be very regular and systematic. They have specificity and determinism and it is often possible to predict exactly which wrong answer is most likely to be given by a particular student. So systematic wrong answers given by a student often provide clues as to how the student is thinking about a certain class of mathematical problems.

This research hinges on frames (Davis, 1984) and concept image (Tall and Vinner, 1981). Such representations and images are explained in that learners and students do not absorb knowledge exactly in the form they are taught by the teachers. Rather they cognise it and try to fit it in what they already know. They accommodate and assimilate new knowledge to their previous experience. Previous experience provided students with frames for processing knowledge. According to Davis, students use the frame they have to interpret and process knew knowledge.

Assimilating and accommodating new knowledge in old frames might be problematic as the frames might be overstretched to generalise on new platforms they do not apply. Such causes an error, a wrong answer due to an inappropriate framework being used to interpret knowledge.

METHODOLOGY

The research was a qualitative design. About seventy, second year mathematics major students of both sexes were given Grade 12 financial mathematics examinations tasks from previous years. The tasks were collected and analysed for errors. After the tasks were analysed, some student teachers were also interviewed to find out what thin king they held made them to produce wrong answers. 12 students chosen for interview constituted a stratified sample of capability and sex.

Students' response to tasks were first analysed under the categories of correct, partially correct and incorrect as well as not attempted. In particular students' scripts were analysed in greater detail about the errors. They made. The errors were analysed under the categories of conceptual, procedural, arbitrary and careless. However in the analysis, the researchers were open as to the new errors that did not fit into these categories.

Also the errors were discussed in the lens of how students constructed knowledge; knowledge representation structures (Davis, 1984) as well as concept images (Davis, 1984). The interviewees will be transcribed so that the main themes that cause students to have errors can emerge.

DISCUSSIONS AND FINDINGS

We argue that students have errors in financial mathematics because they are not familiar with the mathematical concepts underlying mathematics and connections between them. Such concepts are the percentages, time, geometric sequence including the nth term and the sum of the geometric sequence. While some students understand these underlying concepts, they have problems in the application of the series to a practical context such as financial mathematics. Students also have errors and misconceptions because they do not understand financial mathematics terminology, such as simple interest, compound interest, nominal interest rate, effective interest rate, balance, principal, balance, loan, deposit, annuity and so on. Students also have a great deal of errors because they try to answer the questions by blindly substituting values in financial formula.

Thus students' errors in financial mathematics are mainly conceptual. Students have problems on this topic because they want to work it out procedurally without understanding the mathematics of the geometric sequence on which financial mathematics depends. Given this scenario it is important that once students' errors and misconceptions are identified, then teaching must focus on those errors and misconceptions so that they can be resolved. One way to help students resolve their errors and misconceptions is through introducing cognitive conflict. We argue that teaching that does not start with diagnosing students errors and misconceptions in a certain topic is one of the reasons why teaching can be a sterile activity, as it does not connect with the knowledge representation structures that mathematics students have. Helping student teachers to relive the misconceptions they have in mathematics and financial mathematics goes a long way in ensuring that misconceptions in mathematics are not passed from generation to generation.

In distinguishing between student teachers' errors and misconceptions in financial mathematics it is our purpose to highlight the connections between students' errors and misconceptions by pointing out that the errors that students' exhibit in financial mathematics are an outgrowth of consistent and often understandable misconceptions; which are the underlying cognitive representation structures responsible for errors. Besides providing a map of how students' unseen misconceptions result in errors, we assess the extent to which underlying misconceptions lead to the difficulties students face in learning mathematics.

The article is structured as follows: After giving an overview of literature on errors and misconceptions in mathematics, we outline the purpose of the research and the research questions. Next we provide the theoretical framework for the research and the research methodology. Finally, in the last two sections we analyse the data on student errors in financial mathematics tasks and the data from interviews with selected students and conclude with a discussion of our findings.

The study noted that students do not understand the meaning of financial mathematics terms such as compound interest, effective interest rate or annuity. Also students' misunderstanding of the geometric sequence and its inherent application to financial mathematics resulted in many errors. Blind substitution into financial mathematics formulae was common.

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